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Primordial Black Hole Dark Matter: LISA Serendipity

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There has recently been renewed interest in the possibility that the dark matter in the Universe consists of primordial black holes (PBHs). Current observational constraints leave only a few PBH mass ranges for this possibility. One of them is around $10^{-12} M_\odot$. If PBHs with this mass are formed due to an enhanced scalar-perturbation amplitude, their formation is inevitably accompanied by the generation of gravitational waves (GWs) with frequency peaked in the mHz range, precisely around the maximum sensitivity of the LISA mission. We show that, if these primordial black holes are the dark matter, LISA will be able to detect the associated GW power spectrum. Although the GW source signal is intrinsically non-Gaussian, the signal measured by LISA is a sum of the signal from a large number of independent sources suppressing the non-Gaussianity at detection to an unobservable level. We also discuss the effect of the GW propagation in the perturbed Universe. PBH dark matter generically leads to a detectable, purely isotropic, Gaussian and unpolarized GW signal, a prediction that is testable with LISA.

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Introduction.—The existence and the nature of dark matter remains one of the main puzzles in physics [1]. The recent detection of GWs generated by the merging of two $\sim 30 M_\odot$ black holes [2] has renewed the interest in the possibility that all (or a significant part of) the dark matter of the Universe is in the form of primordial black holes (PBHs) (see Refs. [3–6] for recent literature).

A standard way to generate PBHs in the early Universe is to enhance the power spectrum of the comoving curvature perturbation ζ during inflation, but only on scales much smaller than those constrained to be small by CMB observations [7–9] (see Ref. [10] in the case in which standard model Higgs perturbations are used). After reheating, the perturbations are transferred to the radiation, forming PBHs upon horizon reentry if the perturbations are large enough. A region typically collapses to a PBH at horizon entry if the comoving density contrast during radiation domination $\Delta(\vec{x}) = 4\nabla^2\zeta(\vec{x})/(9a^2H^2)$ is larger than a critical value Δ_c (here a is the scale factor and H the Hubble parameter).

We define the comoving curvature perturbation power spectrum as

$$\langle \zeta(\vec{k}_1)\zeta(\vec{k}_2) \rangle' = \frac{2\pi^2}{k_1^3} \mathcal{P}_\zeta(k_1), \quad (1)$$

where we have adopted the standard prime notation indicating that we do not explicitly write down the $(2\pi)^3$

times the Dirac delta of momentum conservation. It is convenient to define the variance of $\Delta(\vec{x})$ as

$$\sigma_\Delta^2(M) = \int_0^\infty d \ln k W^2(k, R_H) \mathcal{P}_\Delta(k), \quad (2)$$

where we have made use of a (Gaussian) window function $W(k, R_H)$ to smooth out $\Delta(\vec{x})$ on the comoving horizon length $R_H \sim 1/aH$ and $\mathcal{P}_\Delta(k) = (4k^2/9a^2H^2)^2 \mathcal{P}_\zeta(k)$. Assuming Gaussian primordial perturbations, the mass fraction β_M of the Universe which ends up in PBHs at the time of formation is approximately (for the non-Gaussian extension see Ref. [11])

$$\beta_M = \int_{\Delta_c}^\infty \frac{d\Delta}{\sqrt{2\pi}\sigma_\Delta} e^{-\Delta^2/2\sigma_\Delta^2} \simeq \frac{\sigma_\Delta}{\Delta_c \sqrt{2\pi}} e^{-\Delta_c^2/2\sigma_\Delta^2}. \quad (3)$$

This corresponds to a present fraction of dark matter $f_{\text{PBH}}(M) \equiv d(\rho_{\text{PBH}}/\rho_{\text{DM}})/d \ln M$ in the form of PBHs of mass M [5]

$$f_{\text{PBH}}(M) \simeq \left(\frac{\beta_M}{6 \times 10^{-9}} \right) \left(\frac{\gamma}{0.2} \right)^{\frac{1}{2}} \left(\frac{106.75}{g_*} \right)^{\frac{1}{4}} \left(\frac{M_\odot}{M} \right)^{\frac{1}{2}}, \quad (4)$$

for a dark matter density parameter today $\Omega_{\text{DM}} h^2 \approx 0.12$. Here $\gamma < 1$ accounts for the efficiency of the collapse and g_* is the number of effective relativistic degrees of freedom (d.o.f.) at horizon entry. We will take $\gamma \simeq 0.2$ [12].

The key point is that, if there are large gradients in the curvature perturbations (for example generated during the last stages of inflation), they inevitably act as a (second-order) source [13–16] of primordial GWs [17,18]. We can relate the mass M of a PBH to the peak frequency of the GWs (not far from the peak frequency of the corresponding curvature perturbations which collapse to form a PBH at horizon entry, $k = 2\pi f = aH$) [18]

$$M \simeq 33\gamma \left(\frac{10^{-9} \text{ Hz}}{f} \right)^2 M_{\odot}. \quad (5)$$

This shows that the mass corresponding to the frequency where the Laser Interferometer Space Antenna (LISA) project [19] has the maximum sensitivity, $f_{\text{LISA}} \simeq 3.4 \text{ mHz}$, is $M \simeq 10^{-12} M_{\odot}$.

The serendipity is that around this mass current observational constraints on the PBH abundances are basically absent [20], thus allowing $f_{\text{PBH}}(M) \simeq 1$, see Fig. 1. Indeed, the Subaru HSC microlensing measurements [21] must be cut around $10^{-11} M_{\odot}$, since below this mass the geometric optics approximation is no longer valid [20,22]: the angular Einstein radius becomes much smaller than the angular size of the star, and the magnification is then too small to be detected [20,22]. Neutron star limits [23] are also not included as they depend on rather controversial assumptions about the dark matter density in globular clusters [20]. The curious reader can find a more expanded discussion in Appendix A of Ref. [24].

It is an exciting coincidence that the optimal frequency range for the LISA observatory corresponds to the mass range where PBHs can account for all the dark matter. In this Letter we show that, if dark matter is composed of PBHs of masses around $10^{-12} M_{\odot}$, then LISA will measure the power spectrum of GWs inevitably associated with the production of the PBHs. Furthermore, and despite the fact

that the generated GWs are intrinsically non-Gaussian (their small-scale source is second-order in the curvature perturbation), we show that the signal measured by LISA would be highly Gaussian. This is because, as with other cosmological GW signals, a very large number of Hubble patches are observed over the resolution area of LISA, giving strong central limit theorem Gaussianization [26]. We also comment on GW propagation in the perturbed Universe and primordial non-Gaussianity, neither of which affect the conclusion.

This Letter contains only the main results; the reader can find the technical details in Ref. [24].

PBHs as dark matter.—From Eq. (4) we see that PBHs of mass $\sim 10^{-12} M_{\odot}$ will form all of the dark matter if their corresponding mass fraction is $\beta_M \sim 6 \times 10^{-15}$. As a benchmark example, we take the comoving curvature perturbation power spectrum (augmented by the standard flat spectrum on large CMB scales) to be the limiting case of a Dirac delta function

$$\mathcal{P}_{\zeta}(k) = A_s k_{\star} \delta(k - k_{\star}). \quad (6)$$

Assuming this spectrum has the huge advantage that we can perform all the calculations analytically. Figure 1 shows the corresponding abundance of PBHs for a representative choice of parameters. We take $k_{\star} R_H \simeq 1$ and $\Delta_c \simeq 0.45$. The precise value of the threshold depends on the shape of the power spectrum [27], but this does not much alter the value of the spectrum amplitude A_s , which is the most relevant quantity for the amplitude of GWs produced. The value of A_s does depend on our assumption of Gaussian perturbations, which may well not be accurately valid since $A_s \sim 0.03$ is quite large. However, even if positive skewness of the Δ distribution meant that $f_{\text{PBH}} \sim 1$ could be obtained with a lower A_s , so that Δ_c was then several more standard-deviation units away from zero, the required variance (proportional to A_s) would only change by an order unity factor (compared to the $\sim \mathcal{O}(100)$ reduction that would be required for the GW signal to become undetectable), so our conclusion should remain robust.

The power spectrum of GWs.—We define the Newtonian-gauge scalar metric perturbation Ψ and the transverse-traceless tensor metric perturbation h_{ij} so that the linearized line element in tightly coupled radiation domination is

$$ds^2 = a^2 \left\{ -(1 + 2\Psi)d\eta^2 + \left[(1 - 2\Psi)\delta_{ij} + \frac{h_{ij}}{2} \right] dx^i dx^j \right\}. \quad (7)$$

We neglect the rare areas of strongly nonlinear GW production associated directly with PBH formation and evolution, and focus on the signal sourced everywhere by second-order combinations of the linear scalar

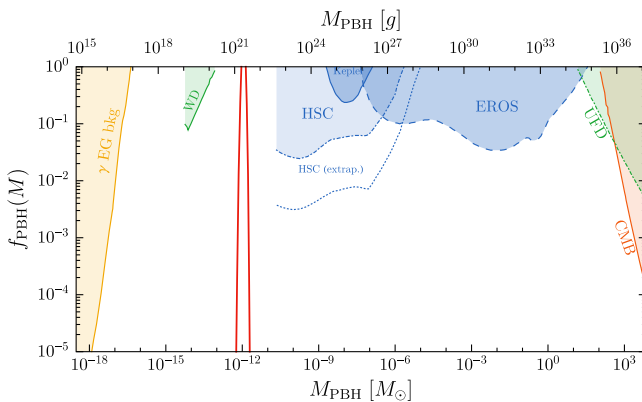


FIG. 1. Current experimental constraints on monochromatic spectra of PBHs at various masses (from Ref. [25] and references therein). The PBH abundance shown as the red line (corresponding to all of the dark matter) has been obtained for $A_s = 0.033$ and $k_{\star} = 2\pi f_{\text{LISA}}$ in Eq. (6).

perturbations. The equation of motion for the GWs is then obtained by expanding Einstein's equations up to second order in the linear perturbations

$$h''_{ij} + 2\mathcal{H}h'_{ij} - \nabla^2 h_{ij} = -4\mathcal{T}_{ij}{}^{\ell m} \mathcal{S}_{\ell m}, \quad (8)$$

where $'$ is the derivative with respect to the conformal time η , $\mathcal{H} = a'/a$ is the conformal Hubble parameter, and $\mathcal{T}_{ij}{}^{\ell m}$ projects the source term $\mathcal{S}_{\ell m}$ into its transverse and traceless part. In the radiation phase the source is given by [13]

$$\mathcal{S}_{ij} = 2\partial_i \partial_j (\Psi^2) - 2\partial_i \Psi \partial_j \Psi - \partial_i \left(\frac{\Psi'}{\mathcal{H}} + \Psi \right) \partial_j \left(\frac{\Psi'}{\mathcal{H}} + \Psi \right). \quad (9)$$

Since this is second order in the perturbations, the sourced GWs are intrinsically non-Gaussian. The source is also local, depending only on spatial derivatives of the perturbations, so the resulting bispectrum will peak in momentum-space configurations where the wave vectors have similar amplitude (no squeezed component). We define the projector in Fourier space using the chiral basis

$$\tilde{\mathcal{T}}_{ij}{}^{\ell m}(\vec{k}) = e_{ij}^{L\ell m}(\vec{k}) \otimes e^{L\ell m}(\vec{k}) + e_{ij}^{R\ell m}(\vec{k}) \otimes e^{R\ell m}(\vec{k}), \quad (10)$$

where $e_{ij}^{L,R}$ are the polarization tensors. In Eq. (9) the scalar perturbation $\Psi(\eta, \vec{k})$ can be written in terms of the initial gauge-invariant comoving curvature perturbation as [28]

$$\Psi(\eta, \vec{k}) \equiv \frac{2}{3} T(k\eta) \zeta(\vec{k}), \quad (11)$$

where the transfer function during radiation domination with constant d.o.f. is $T(x) = (9/x^2)[\sin(x/\sqrt{3})/(x/\sqrt{3}) - \cos(x/\sqrt{3})]$. A straightforward calculation approximating the primordial perturbations as Gaussian leads to the current abundance of GWs [29]

$$\frac{\Omega_{\text{GW}}(f)}{\Omega_{r,0}} = \frac{c_g}{72} \int_{-\frac{1}{\sqrt{3}}}^{\frac{1}{\sqrt{3}}} dd \int_{\frac{1}{\sqrt{3}}}^{\infty} ds \left(\frac{(d^2 - 1/3)(s^2 - 1/3)}{s^2 - d^2} \right)^2 \times \mathcal{P}_{\zeta} \left(\frac{k\sqrt{3}}{2}(s+d) \right) \mathcal{P}_{\zeta} \left(\frac{k\sqrt{3}}{2}(s-d) \right) \mathcal{I}^2(d, s), \quad (12)$$

where $k = 2\pi f$, $\Omega_{r,0}$ parameterizes the current density of radiation if the neutrinos were massless, $c_g \simeq 0.4$ accounts for the change of the effective d.o.f. of the thermal radiation during the evolution (assuming standard model physics), $\mathcal{I}^2 \equiv \mathcal{I}_c^2 + \mathcal{I}_s^2$, and

$$\mathcal{I}_c(x, y) = 4 \int_0^\infty d\tau \tau (-\sin \tau) [2T(x\tau)T(y\tau) + (T(x\tau) + x\tau T'(x\tau))(T(y\tau) + y\tau T'(y\tau))], \quad (13)$$

$\mathcal{I}_s(x, y)$ being the same function, but with $\sin \tau$ replaced by $(-\cos \tau)$, see Ref. [30]. For the monochromatic power spectrum [Eq. (6)] we obtain (see also Refs. [15, 17, 30])

$$\frac{\Omega_{\text{GW}}(f)}{\Omega_{r,0}} = \frac{A_s^2 c_g f^2}{15552 f_\star^2} \left(\frac{4f_\star^2}{f^2} - 1 \right)^2 \theta \left(2 - \frac{f}{f_\star} \right) \mathcal{I}^2 \left(\frac{f_\star}{f}, \frac{f_\star}{f} \right), \quad (14)$$

where $f_\star = k_\star/2\pi$ and $\theta(x)$ is the step function. The current abundance of GWs is given in Fig. 2 with $k_\star \sim k_{\text{LISA}} = 2\pi f_{\text{LISA}}$ and $A_s \sim 0.033$. Since the result is only a function of f/f_\star , for other possible f_\star (with typical black hole masses as indicated on the top axis) the predicted spectrum simply shifts sideways in f . This shows that, if PBHs of masses in the range $10^{-15} M_\odot \lesssim M \lesssim 10^{-11} M_\odot$ form the dark matter (or even a fraction of it), LISA will measure the GWs popping out during the PBH formation time.

The primordial bispectrum of GWs.—Since the GW source is nonlinear, the three-point correlator of the GWs is not vanishing. Its computation is straightforward in the approximation of Gaussian initial perturbations [29]

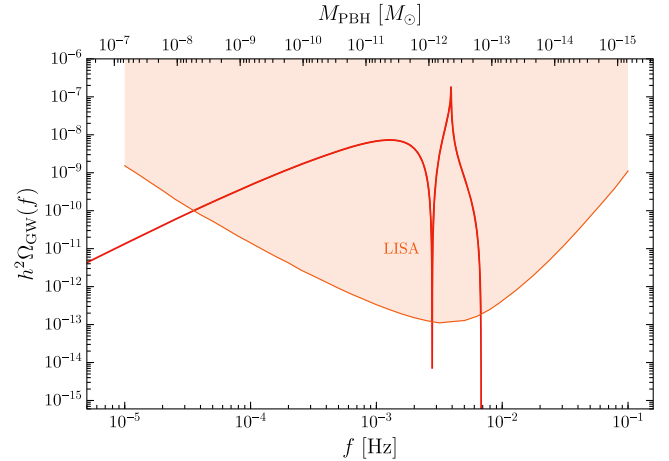


FIG. 2. The power spectrum of GWs generated by PBHs compared with the power-law integrated sensitivity for LISA estimated on the basis of the proposal [19]: the proposed design (4yr, 2.5 Gm of length, 6 links) is anticipated to have a sensitivity in between those called C1 and C2 in Ref. [31]. The spike is due to the trigonometric functions coming from the radiation transfer functions in \mathcal{I}^2 , giving a resonant effect at $f \sim 2f_{\text{LISA}}/\sqrt{3}$, as explained in Ref. [15]. The spike and slow falloff in power to low frequencies are an artifact of assuming a monochromatic power spectrum; physical spectra would typically give a smooth spectrum with white noise ($\propto f^3$) at low frequencies [29], but a similar overall amplitude.

$$\begin{aligned}
 & \langle h_{\lambda_1}(\eta, \vec{k}_1) h_{\lambda_2}(\eta, \vec{k}_2) h_{\lambda_3}(\eta, \vec{k}_3) \rangle' \\
 &= \left(\frac{8\pi}{9} \right)^3 \int d^3 p_1 \frac{1}{k_1^3 k_2^3 k_3^3 \eta^3} \\
 & \times e_{\lambda_1}^*(\vec{k}_1, \vec{p}_1) e_{\lambda_2}^*(\vec{k}_2, \vec{p}_2) e_{\lambda_3}^*(\vec{k}_3, \vec{p}_3) \frac{\mathcal{P}_\zeta(p_1)}{p_1^3} \frac{\mathcal{P}_\zeta(p_2)}{p_2^3} \frac{\mathcal{P}_\zeta(p_3)}{p_3^3} \\
 & \times \left\{ \left[\cos(k_1 \eta) \mathcal{I}_c \left(\frac{p_1}{k_1}, \frac{p_2}{k_1} \right) + \sin(k_1 \eta) \mathcal{I}_s \left(\frac{p_1}{k_1}, \frac{p_2}{k_1} \right) \right] \right. \\
 & \left. \times (1 \rightarrow 2 \text{ and } 2 \rightarrow 3)(1 \rightarrow 3 \text{ and } 2 \rightarrow 1) \right\}, \quad (15)
 \end{aligned}$$

where $\vec{p}_2 = \vec{p}_1 - \vec{k}_1$, $\vec{p}_3 = \vec{p}_1 + \vec{k}_3$, and where $e_{\lambda}^{*ij}(\vec{k}, \vec{p}) = e_{\lambda}^{*ij}(\vec{k}) p_i p_j$ are the polarization tensors and $\lambda = L, R$. The bispectrum of GWs is dominated by the equilateral configuration [24], $k_1 \simeq k_2 \simeq k_3 \equiv k$, as expected since it is sourced by gradients of the curvature perturbations when the latter reenter the horizon. For the equilateral configuration and monochromatic power spectrum [Eq. (6)], the bispectrum today at time η_0 is

$$\begin{aligned}
 & \langle h_{\lambda_1}(\vec{k}_1) h_{\lambda_2}(\vec{k}_2) h_{\lambda_3}(\vec{k}_3) \rangle'_{\eta_0, \text{EQ}} \\
 &= \left(\frac{A_s a_f}{k^2 k_* \eta_f} \right)^3 \frac{1024 \pi^3}{729} \\
 & \times \left| \frac{1}{\sqrt{2}} \mathcal{I} \left(\frac{k_*}{k}, \frac{k_*}{k} \right) \right|^3 \frac{\theta(\sqrt{3}k_* - k)}{\sqrt{3}k_*^2/k^2 - 1} \mathcal{D}_{\lambda_1 \lambda_2 \lambda_3} \left(\frac{k_*}{k} \right), \quad (16)
 \end{aligned}$$

where η_f is a time well after the modes have entered the horizon but before the thermal d.o.f. change (we have also taken $a_0 = 1$). The function $\mathcal{D}_{\lambda_1 \lambda_2 \lambda_3}(x) = 365/6912 - 61x^2/192 + 9x^4/16 - x^6/4$ for the RRR and LLL polarizations and $\mathcal{D}_{\lambda_1 \lambda_2 \lambda_3}(x) = x^6(-4 + 1/x^2)^2(-12 + 5/x^2)^2/768$ for the other combinations, see Fig. 3. Note that in Eq. (16) we have dropped the phases of the bispectrum. This is a crucial point when asking if the bispectrum can be observed by LISA.

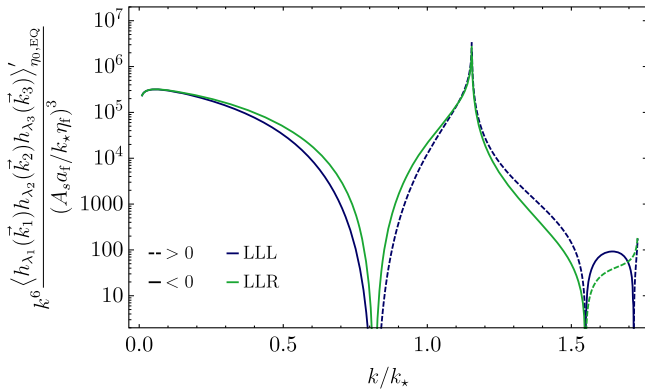


FIG. 3. The normalized shapes for the various primordial bispectra in the equilateral configurations.

Tensor non-Gaussianity is not locally observable.—The bispectrum calculated above is defined over a constant time hypersurface and hence not directly locally observable. Is tensor non-Gaussianity actually measurable by LISA? As explained previously, the answer is no because the light cone includes signals generated in a large number ($\sim 10^{40}$) of independent Hubble patches, and hence the observed signal should sum to be highly Gaussian. How this is consistent with the bispectrum calculation has been a cause of some confusion, which we clarify here.

Non-Gaussianity is present when there are correlations between different Fourier modes, so non-Gaussianity is synonymous with “phase correlations” [32]. For example, correlations between three approximately equal wave numbers forming a positive bispectrum triangle correspond to the signal being relatively more concentrated at real spaces’ peaks. However, at the time and location of observation, the phases are almost completely uncorrelated, as we now explain.

LISA measures the effect of a collection of GWs arriving at the detector from all possible directions, not a single one. Consider the change in the light-travel time for a photon emitted from \vec{x}_1 at one end of the detector at time η_0 and arriving at the other end of the arm (at relative position \vec{L}) due to a passing gravitational wave. Integrating the strain along the photon path gives [33]

$$\begin{aligned}
 \Delta\eta(\eta_0) &= \frac{L}{2} \int \frac{d^3 k}{(2\pi)^3} e^{i\vec{k} \cdot \vec{x}_1} \sum_{\lambda} e_{\lambda}(\vec{k}, \hat{L}) [e^{ik\eta_0} h_{\lambda}(\vec{k}) \mathcal{M}(\hat{L} \cdot \hat{k}, k) \\
 &+ e^{-ik\eta_0} h_{\lambda}^*(-\vec{k}) \mathcal{M}^*(-\hat{L} \cdot \hat{k}, k)], \quad (17)
 \end{aligned}$$

where

$$\mathcal{M}(\hat{L} \cdot \hat{k}, k) = e^{ikL(1 - \hat{L} \cdot \hat{k})/2} \text{sinc} \left[\frac{kL(1 - \hat{L} \cdot \hat{k})}{2} \right] \quad (18)$$

(summing the time for the light to come back to the initial point does not change the argument). The bispectrum of the $\Delta\eta$ measured in three arms at locations \vec{x}_i will be proportional to objects like

$$\prod_{i=1}^3 \int d^3 k_i e^{i\vec{x}_i \cdot \vec{k}_i} e^{\pm i k_i \eta_0} \mathcal{M}(\hat{L}_i \cdot \hat{k}_i, k_i) \langle h_{\lambda_1}(\vec{k}_1) h_{\lambda_2}(\vec{k}_2) h_{\lambda_3}(\vec{k}_3) \rangle. \quad (19)$$

Since the bispectrum is peaked around some momentum $k_* \gg \eta_0^{-1}$, and \mathcal{M} varies slowly relative to the rapidly oscillating terms $\exp(i \sum_i \pm k_i \eta_0)$, these integrals average to zero except where $\sum_i \pm k_i = 0$ because of the delta function in the three-point correlation. Note that the power spectrum is not affected since the corresponding phases cancel.

The nonzero contribution from $\sum_i \pm k_i = 0$, corresponding to all three \vec{k} wave vectors being aligned, is the in-principle observable signal from correlating three wavelengths emitted by the same Hubble patch in a particular direction. However, the alignment of directions for $\sum_i \vec{k}_i = 0$ to imply $|\sum_i \pm k_i \eta_0| \lesssim 1$ has to be very precise, with angular precision $\delta\theta \sim 1/\sqrt{k\eta_0}$, and hence contributes to a negligible fraction $\sim (k\eta_0)^{-1} \ll 1$ of the integral over all angles. The number of independent Hubble patches contributing to the signal is $N \propto (k\eta_0)^2$, so the $\sim (k\eta_0)^{-1} \propto 1/\sqrt{N}$ scaling is just the one expected when measuring the bispectrum of the sum of N independent signals. This is a consequence of the central limit theorem: within the measurement time, the detector does not measure a single wave, but a sum of GWs within a given momentum width from all directions, so the signal is strongly Gaussianized.

Could the signal be measured by collecting observations over a long observation time $\Delta\eta$? The phases will average if the range of k values included in the observed band is large compared to $1/\eta_0$. In a total observation time $\Delta\eta$, the bandwidth that is in principle resolvable is $\sim 1/\Delta\eta$. However, since $\Delta\eta \ll \eta_0$ for any observation time small compared to the age of the Universe, the signal would still be lost.

How about building a large array of LISA-like detectors? In principle a large array could capture the wave front from each source horizon volume at multiple locations today, giving observable phase correlations. The array would however have to be cosmologically large to capture more than a tiny fraction of the signal, since the correlated shells of GW emission have a radius of η_0 today. As the array is made larger, the phases also decorrelate for other reasons, for example due to variations in the Shapiro time delay as the waves propagate across the inhomogeneous Universe. This randomizes the GW phases, and suppresses the GW bispectrum.

To demonstrate this effect, consider the propagation of the GW through the inhomogeneous Universe. We can work in the geometrical optic limit where the wavelength of the GW is much smaller than the size of the gravitational potentials (though microlensing events could also be relevant). The Shapiro time delay is given by the integral along the GW path of the potential, with total delay at position \vec{x} for a GW observed in direction $-\hat{k}$

$$\delta\eta(\vec{x}) = 2 \int_{\eta_e}^{\eta_0} d\eta' \Psi(\vec{x} + \hat{k}(\eta' - \eta_0), \eta'), \quad (20)$$

where η_e is the emission time (which we can take to be zero). When received, the GW has therefore acquired a phase shift of $k\delta\eta$ compared to the propagation in a homogeneous Universe. This is not a problem, as long as the phase shift is the same for all the GW measurements (it would just change the overall phase). However, if they vary, the average correlation of waves at three points would pick up a factor

of $\langle \exp(i\sum_i k_i \delta\eta_i) \rangle$. Averaging the exponentials assuming Gaussian $\delta\eta_i$ gives an exponential suppression of the bispectrum with a product of terms of the form $e^{-k_i k_j C_{ij}/2}$, where C_{ij} is a correlation of a time delay difference between two of the measurements. For cosmologically separated observation points, the suppression eventually becomes $\sim e^{-\sum_i k_i^2 \sigma^2/2}$ [24], where $\sigma = \langle \delta\eta^2 \rangle^{1/2} \approx 10^{-4} \eta_0$ [34], which wipes out the signal since $k\eta_0 \sim 10^{16}$.

The only remaining way that there could be observable non-Gaussianity in the GW distribution is if there were long-range correlations between Hubble patches at the time of GW emission, for example due to squeezed non-Gaussianity of the primordial perturbation modulating the local fluctuation amplitude. This could in principle lead to the observed GW power varying over observably large angular scales. However, if the dark matter is PBHs, the abundance of PBHs is very sensitive to the amplitude of perturbations, and would vary spatially if there were long-range variations of the local power spectrum amplitude. The observed large-scale homogeneity of the dark matter density (absence of CDM isocurvature modes) therefore also rules out this option at any significant level. The prediction of a purely Gaussian isotropic gravitational wave background associated with PBH formation is robust. The quadrupole sources in each independent horizon volume also have uncorrelated orientations (unless there is large anisotropic squeezed primordial non-Gaussianity), so the observed sum of the signals from many volumes is also expected to be unpolarized to high accuracy.

Conclusions.—If most (if not all) of dark matter is composed of PBHs, this is a very economical option since no physics beyond the standard model is required. If the PBHs forming the dark matter have a mass of the order of $10^{-12} M_\odot$ this scenario is still observationally viable, and also testable since it inevitably produces a background of gravitational waves that would be detectable by LISA. Although the gravitational wave source is intrinsically non-Gaussian, the observed signal today should be isotropic and Gaussian. The task of distinguishing the signal from that from phase transitions or inflationary sources (which also generically predict isotropic Gaussian backgrounds) must then rely on detailed study of the power spectrum shape, a topic that deserves further study. If new more robust constraints on f_{PBH} appear, they can be satisfied by decreasing A_s by a small amount (since f_{PBH} is exponentially sensitive to A_s), which could still leave a potentially detectable GW signal associated with a smaller PBH fraction. Finally, the GW signal associated with a narrow mass range of PBHs is peaked in frequency, and hence should be distinguishable from a nonprimordial stochastic GW background from astrophysical sources characterized by approximately power-law spectra, such as the signal from black hole mergers.

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